Notes

Linear Functions

- Equation of a line with gradient m and vertical intercept c is $y = mx + c$.
- Equation of a line with gradient m and passing through the point (h, k) is $y - k = m(x - h)$.
- Two lines with gradients m_1 and m_2 respectively are
  (i) parallel if $m_1 = m_2$.
  (ii) perpendicular if $m_1 \times m_2 = -1$.

Quadratic Functions

- For the parabola $y = a(x - h)^2 + k$: turning point is (h, k).
- For the parabola $y = ax^2 + bx + c$: line of symmetry is $x = -\dfrac{b}{2a}$. (this is the x-coordinate of the turning point)
- For the parabola $y = a(x - p)(x - q)$: line of symmetry is $x = \dfrac{p+q}{2}$. (this is the x-coordinate of the turning point)
- Parabola has a minimum point if the coefficient of the x^2 term is positive, maximum otherwise.
- For the equation $ax^2 + bx + c = 0$ roots are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
  Roots are real and different if $b^2 > 4ac$.
  Roots are real and repeated if $b^2 = 4ac$.
  Roots are complex if $b^2 < 4ac$.

Cubics

- For the cubic $y = ax^3 + bx^2 + cx + d$:

Factors of $ax^3 + bx^2 + cx + d$	No. of roots
3 distinct linear factors	3
3 linear factors with 2 the same	2
all 3 linear factors the same	1
1 linear and 1 non-reducible quadratic	1

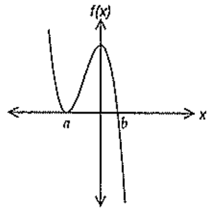
Rectangular Hyperbola

$y = \dfrac{k}{x-a}$ has:

- a horizontal asymptote with equation $y = 0$.
- a vertical asymptote with equation $x = a$.

Polynomials

- To find the equation of the given curve: use roots with a multiplier k.
  $y = k(x - a)^2(x - b)$
  Root is repeated when the curve bounces off the x-axis at that root.



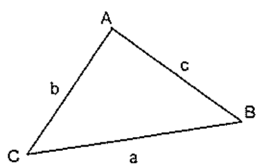
Functions

- A relation r between sets X and Y is a rule that associates (maps) elements in set X with elements in set Y.
- A function f between sets X and Y is a **rule** that associates **each** element in set X with a **unique** element in set Y.
- A function f has either a 1 to 1 rule or a 1 to many rule.
- The graph of function f passes the "vertical line" test'.
-

Functions	Domain	Range
$y = \sqrt{x-a} + b$	$x \geq a$	$y \geq b$
$y = a^x + b$	$\mathbb{R}$	$y > b$
$y = \dfrac{k}{x-a}$	$x \neq a$	$y \neq 0$

- Circles are relations with equations that can be written in the form:
  $(x - a)^2 + (y - b)^2 = r^2$.
  - centre (a, b)
  - radius r.
- For $y = -kf(-ax + b) + m$:
  1. Translate b units left along the x-axis — **T**
  2. Dilate along the x-axis by factor $1/a$ — **D**
  3. Reflect about the y-axis — **R**
  4. Reflect about the x-axis — **R**
  5. Dilate along the y-axis by factor k — **D**
  6. Translate m units up along the y-axis — **T**

Non-Right Triangles

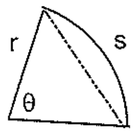


- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$
- $a^2 = b^2 + c^2 - 2bc \cos A$
- $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
- Area of Triangle $= \frac{1}{2} ab \sin C$

Arcs and Sectors

Angle θ is in radians

- Arc length $s = r\theta$
- Area of sector = $\frac{1}{2}r^2\theta$
- Area of segment
  = $\frac{1}{2}r^2(\theta - \sin\theta)$



Exact Values

θ°	θ rad	sin θ	cos θ	tan θ
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	∞

Trigonometric Identities

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

- Trig Graphs

	$y = a\sin(bx + c) + d$ $y = a\cos(bx + c) + d$
Mean Line	$y = d$
Amplitude	$\lvert a\rvert$
Min./Max. y	Min: $d - \lvert a\rvert$, Max: $d + \lvert a\rvert$
Period	$360°/b$ or $2\pi/b$
Phase shift	*Shifted c/b degrees/radians to the left*

	$y = a\tan(bx + c) + d$
Mean Line	$y = d$
Period	$180°/b$ or π/b
Phase shift	*Shifted c/b degrees/radians to the left*

Set Notation

Symbol	Meaning
$\in$	is an element of
$\subset$	is a subset of
$\cap$	intersection
$\cup$	union
$n(A)$ or $\lvert A\rvert$	No. of elements in set A
A' or $\overline{A}$	Complement of A

Combinations

- $^nC_r \equiv \binom{n}{r} \equiv \dfrac{n!}{r!\,(n-r)!}$

  $= \dfrac{\overbrace{n \times (n-1) \times (n-2) \times \ldots\ldots \times (n-r+1)}^{r\ terms}}{r!}$.

- $^nC_r = {}^nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$.
- $\binom{n}{0} = \binom{n}{n} = 1$ and $\binom{n}{1} = \binom{n}{n-1} = n$.
- r items can be chosen from n items all different:
  - without replacement in ^nC_r ways
  - with replacement in n^r ways.
- $(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2$

  $\ldots + \binom{n}{k}x^{n-k}y^k + \ldots$

  $+ \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$

Probability

- $0 \le P(A) \le 1$.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cap B) = P(A) \times P(B|A)$
- $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$
- $P(\overline{A}) = 1 - P(A)$
- Two events A and B are mutually exclusive if $P(A \cap B) = 0$.
- To show that A and B are mutually exclusive, show that $P(A \cap B) = 0$.
- Two events A and B are independent if $P(A|B) = P(A)$ or $P(B) = P(B|A)$.
- To show that A and B are independent:
  - show that $P(A|B) = P(A)$ or $P(B) = P(B|A)$
  - or show that $P(A \cap B) = P(A) \times P(B)$.

Indices

- $a^x \times a^y = a^{x+y}$ $\quad \dfrac{a^x}{a^y} = a^{x-y}$

  $(a^x)^y = a^{xy}$ $\quad a^0 = 1$

  $\dfrac{1}{a^x} = a^{-x}$ $\quad \sqrt[n]{a} = a^{\frac{1}{n}}$

Arithmetic Progression

- General Rule for the nth term:
  $T_n = a + (n-1)d$
- Recursive equation:
  $T_{n+1} = T_n + d \qquad T_1 = a$
- Sum of first n terms:
  $$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

Geometric Progression

- General Rule for nth term: $T_n = a \times r^{n-1}$
- Recursive equation:
  $T_{n+1} = T_n \times r \qquad T_1 = a$
- Sum of first n terms:
  $$S_n = \frac{a(1-r^n)}{1-r} \quad \text{where } r \neq 1$$
- Sum to infinity for $-1 < r < 1$:
  $$S_\infty = \frac{a}{1-r}$$

Exponential Growth and Decay

- $P(t+1) = P(t) \times r$ where $P(0)$ = initial value
- $P(t) = P(0)r^t$

Differentiation

- $f'(x) = \frac{d}{dx}f(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$
- $y = a\,x^n \Rightarrow y' = n \times a\,x^{n-1}$

Rate of change

- The instantaneous rate of change of Q at time $t = a$ is $Q'(a)$.
- The average rate of change between $t = a$ and $t = b$ is $\frac{Q(b) - Q(a)}{b - a}$

Features of graphs

$y = f(x)$	$y = f'(x)$
max point	x-intercept (crosses x-axis from above to below)
min point	x-intercept (crosses x-axis from below to above)
inflection point	turning point

Stationary & Inflection Points

- For max point at $x = a$: $y' = 0$,

x	a^-	a	a^+
y'	+	0	–

- For min point at $x = a$: $y' = 0$,

x	a^-	a	a^+
y'	–	0	+

- For horizontal inflection point at $x = a$:

x	a^-	a	a^+
y'	±	0	±

Integration

- $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \qquad [n \neq -1]$

Rectilinear Motion

- Displacement at time t, $x = \int v\,dt$
  Velocity $v = \frac{dx}{dt} = \int a\,dt$
  Acceleration $a = \frac{dv}{dt}$
- Body changes direction when $v = 0$ and $a \neq 0$.
- Body returns to origin when $x = 0$.

it: